ISSN:0975-9646

Strong Domination Number of Some Graphs in Context of Vertex Switching

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Abstract - **In present paper we have used reconstruction of graphs. A vertex switching I_p of a graph I is attained by taking a vertex p of graph I, removing all edges incident to p and adding edges to every vertex not adjacent to p in I. The resultant graph is called a Switch Card of the original graph. This paper applies the concept of vertex switching to some graphs like Pan graph, Barbell graph, Peterson graph and Sun-let graph and later we compare the strong domination number of the graphs before and after applying vertex switching.**

Keyword: **Strong Domination Number, Pan graph, Barbell graph ($B(p, m)$), Sun-let graph (S_n), Peterson Graph, vertex switching.**

I. INTRODUCTION

We will consider simple, finite and undirected graph throughout the paper. The study of domination in graphs came about partially as a result of the study of games and recreational mathematics. Mathematicians premeditated that how a particular chess piece could be demonstrated on a chess board in such a manner that they would dominate or attack, every square on the board. Dominating sets in graph theory are defined as sets of vertices that are adjacent to all other vertices in the graph, and this property allows them to control the entire network. The concept of edge domination number was first introduced by the mathematician Claude Berge [1] in his paper "Two theorems in graph theory" published in 1958. Vaidya et.al. [2] discussed the domination number of some path graphs. Galby et. al. [3] discussed the concept of reducing domination number of graphs via edge contraction and vertex deletions. Galby et.al. [4] discussed the idea of reducing the domination number of some graphs like P_9-free graphs and Bipartite graphs via edge contractions. Ghodasara and Rokad [5] talk about the cordial labeling of some special graphs in context of vertex switching. Vaidya et.al. [6] presented some cycle related cordial graphs in the context of vertex Switching. For basic terminologies and definitions, we refer Harary [7].

The notation of $V(I)$ represents the set of vertices in the graph and $E(I)$ denotes the set of edges in the graph. The degree of a vertex $'p'$ can be calculated by counting the number of edges that meet at that vertex and it is represented by the notation $\deg(p)$. The strong domination number of a graph is represented by γ_{sdn}.

Definition 1.1
Let $I = (V, E)$ and $p, q \in I(G)$. If $p, q \in E(I)$ and $\deg(p) \geq \deg(q)$ then we say that p strongly dominates q. A subset S of $V(I)$ is called a strong dominating set of I if every $p \in I(G) - S$ is strongly dominated by some $p \in S$.

Definition 1.2
The smallest cardinality of strong dominating set is called a Strong Domination number. It is denoted as γ_{sdn}.

Definition 1.3
A vertex switching I_p of a graph I is obtained by taking a vertex p of graph I, removing all edges incident to p and adding edges to every vertex not adjacent to p in I.

Definition 1.4
The $m-$ pan graph is attained by joining a cycle graph C_m to a singleton graph K_1 with a bridge.

Definition 1.5
The $m-$ barbell graph is the simple graph attained by connecting two copies of a complete graph K_m by a bridge. It is denoted as $B(p, m)$.

Definition 1.6
The Peterson graph is an undirected graph with ten nodes and fifteen links, commonly drawn as pentagram within a pentagon with corresponding vertices attached to each other.

Definition 1.7
The sun-let graph is the graph attained by attaching pendant edges to a m −cycle graph. It is denoted as S_n.

II. MAIN RESULT

Theorem 2.1 Let H be a $m - Pan$ graph then, the strong domination number of H is given by-

$$\gamma_{sdn} = \begin{cases} \mathbb{N}; & when\ m \equiv 0\ (mod\ 3) \\ \mathbb{N} - \{1\}; & when\ m \equiv 1, 2\ (mod\ 3) \end{cases}$$

Proof: Let H be a $m - Pan$ graph and let $\{p_1, p_2, p_3, \dots \dots, p_m\}$ be the successive vertices of H. The labeling begins with the singleton vertex in the anti – clockwise direction.

Let $V = \{p_1, p_2, p_3, \dots \dots, p_m\}$ be the set of vertices and let D be a dominating set of H. Since every vertex in H is adjacent to two other vertices in the graph and degree of each vertex is greater than equal to one.

The set D includes those vertices that have highest degree among them and are adjacent to each other while their adjacent vertices are not included in the set.

Case 1: When $m \equiv 0\ (mod\ 3)$
When m is divisible by 3, then number of vertices present in the dominating set D will start from 1 and increase by one in every subsequent step i.e. set of $\mathbb{N}$.
Case 2: When $m \equiv 1,2\ (mod\ 3)$
When m is not divisible by 3, then number of vertices present in D will start from 2 and augment by one in every subsequent step i.e. $\mathbb{N} - \{1\}$.
Hence, the theorem.
Example 2.1: The Strong Domination Number of $6 - Pan$ graph is 2.

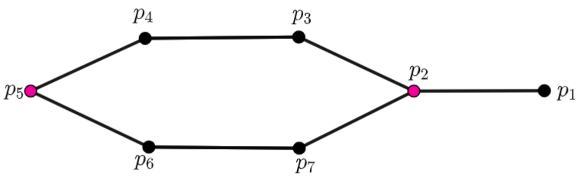
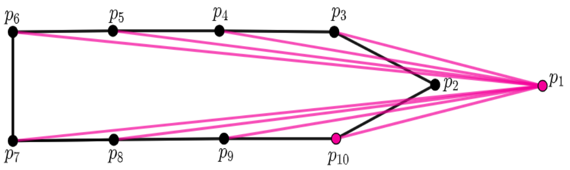


Fig 1: $6 - Pan$ graph

Theorem 2.2 Let H be a $m - Pan$ graph, then the Strong Domination number of H in context of vertex switching is-

$$\gamma_{sdn} = \begin{cases} 1, & when\ m = 3 \\ 2, & when\ m > 3 \end{cases}$$

Proof: Let H be a $m - Pan$ graph and let $\{p_1, p_2, p_3, \dots \dots, p_m\}$ be the successive vertices of H. Here, H_p signifies the vertex switching of H with respect to the vertex p of H. The labeling begins from the switched vertex, considered as p_1 and proceed in an anti-clockwise direction to label the remaining vertices. The following cases will occur –
Case 1: When $m = 3$
By the definition of vertex switching, the vertex p_1 will get attach to all other vertices except p_2. Therefore, the dominating set will have only one element whose degree is maximum i.e. $\gamma_{sdn} = 1$.
Case 2: When $m > 3$
As the vertex p_1 is connected to all other vertices of H except p_2. Therefore, the dominating set of H will always have two elements whose degree is maximum and are adjacent to all other vertices i.e. $\gamma_{sdn} = 2$.
Hence, the theorem.
Example 2.2: The Strong Domination Number of $9 - Pan$ graph in context of vertex switching is 2.



Fi 2: $9 - Pan$ graph in context of vertex switching

Theorem 2.3 Let H be a $m - Barbell$ graph, then the Strong Domination Number of H is 2.
Proof: Let H be a $m - Barbell$ graph and let $\{p_1, p_2, p_3, \dots \dots, p_m\}$ be the vertices of H. The labelling will start from the first complete graph in the anti – clockwise direction.
Let $V = \{p_1, p_2, p_3, \dots \dots, p_m\}$ be the set of vertices and let D be a dominating set of H. As Barbell graph is obtained by connecting two copies of complete graph which implies that the degree of each vertex of $m - Barbell$ graph is at least $m - 1$. Therefore, there exist only two vertices which are adjacent to all other vertices of H and have highest degree while their adjacent vertices are not included.
Hence, the Strong Domination Number of $m - Barbell$ graph is 2.
Example 2.3 The Strong Domination Number of $6 - Barbell$ graph is 2.

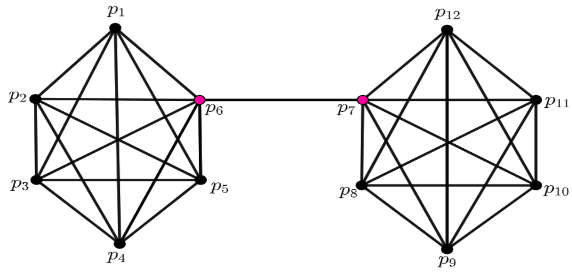


Fig 3: $6 - Barbell$ graph

Theorem 2.4 Let H be a $m - Barbell$ graph, then the Strong Domination Number of H after vertex switching is as follows –

$$\gamma_{sdn} = \begin{cases} 2, & 3 \le m \le 8 \\ 3, & m \ge 9\ and\ m\ is\ odd \\ 2, & m > 9\ and\ n\ is\ even \end{cases}$$

Proof: Let H be a $m - Barbell$ graph and let $\{p_1, p_2, p_3, \dots \dots, p_m\}$ be the successive vertices of H. Here, H_p signifies the vertex switching of H with respect to the vertex p of H. The labeling begins from the switched vertex, considered as p_1 and proceed in an anti-clockwise direction to label the remaining vertices. Let $V = \{p_1, p_2, p_3, \dots \dots, p_m\}$ be the set of vertices and let D be a dominating set of H. The following cases will occur-
Case 1: When $3 \le m \le 8$
As the vertex p_1 is the switched vertex, so p_1 will be cut off from all the vertices of first complete graph and will be bridged to all the vertices of the other complete graph. In this case, we will always have two vertices which will dominate the complete graph and allocated to the set D. Hence, the Domination number of H in context of vertex switching is 2.
Case 2: When $m \ge 9$ and m is odd
As the vertex p_1 is the switched vertex, so p_1 will be cut off from all the vertices of first complete graph and will be bridged to all the vertices of the other complete graph. In this case, there are two vertices which dominate the first complete graph and the vertex p_1 will dominate the other complete graph. Hence, the Domination number of H in the context of vertex switching is 3.
Case 3: When $m > 9$ and m is even
As the vertex p_1 is the switched vertex, so p_1 will be cut off from all the vertices of first complete graph and will be bridged to all the vertices of the other complete graph. In this case, there is one vertex which dominates the first

complete graph and the vertex p_1 will dominate the other complete graph. Hence, the Domination number of H in the context of vertex switching is 2.
Hence, the theorem.

Example 2.4 The Strong Domination Number of $12 - Barbell$ graph in context of vertex switching is 2.

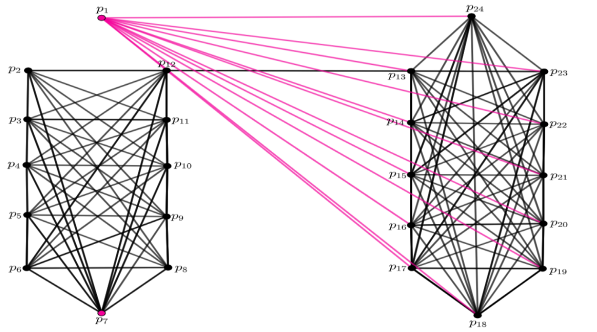


Fig 4: $12 - Barbell$ graph in context of vertex switching

Theorem 2.5 The strong domination number of $m - sunlet$ graph is m.
Proof: Let H be a $m - sunlet$ graph and let $\{p_1, p_2, p_3, \dots \dots, p_m\}$ be the vertices of H. The labelling will start from any vertex of the cycle and then the pendent vertices. Let D be the dominating set of H. All the elements in the set D are vertices from the cycle graph that are connected solely to the pendent vertices which is m. Hence, the Strong Domination number of H is m.

Example 2.5 The strong domination number of 6-sunlet graph is 6

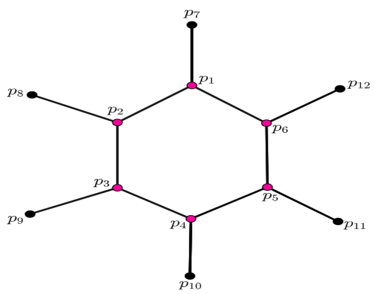


Fig 5: $6 - sunlet$ graph

Theorem 2.6 The strong domination number of m −sunlet graph in context of vertex switching is 2.
Proof: Let H be a $m - Pan$ graph and let $\{p_1, p_2, p_3, \dots \dots, p_m\}$ be the successive vertices of H. Here, H_p denotes the vertex switching of H with respect to the vertex p of H. The labeling begins from the switched vertex, considered as p_1 and proceed in an any direction to label the remaining vertices.
Let $V = \{p_1, p_2, p_3, \dots \dots, p_m\}$ be the set of vertices and let D be a dominating set of H. As the vertex p_1 is the switched vertex, so p_1 will be cut off from its adjacent vertex and get attached to all other vertices of the graph H. Here in every case, the set D will include only two vertices of the graph H excluding the adjacent vertices of it. Hence, the Strong Domination Number of $m - sunlet$ graph in context of vertex switching is 2.

Example 2.6 The strong domination number of $8 - sunlet$ graph in context of vertex switching is 2.

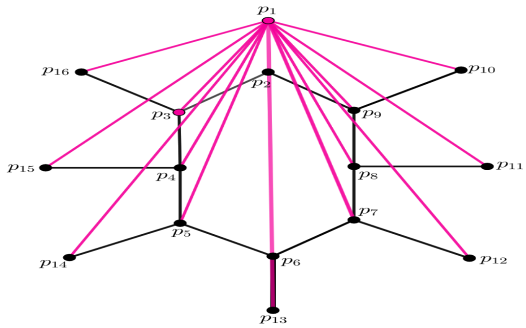


Fig 6: $8 - sunlet$ graph in context of vertex switching

Theorem 2.7 Let H be Peterson graph, then the strong domination number of H is 3.
Proof: Let H be a Peterson graph which is an undirected graph featuring 10 vertices and 15 edges. Let $\{p_1, p_2, p_3, \dots \dots, p_m\}$ be the vertices of H and let D be the dominating set of H. The set D comprises those vertices of H which are adjacent to all other vertices of H and have highest degree while their adjacent vertices are not included. The number of vertices in the set D is 3. Hence, the Strong Domination Number of H is 3.
Example 2.7 The Strong Domination Number of Peterson graph is 3.

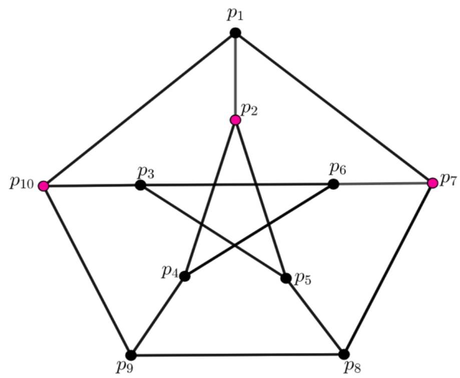


Fig 7: Peterson graph

Theorem 2.8 Let H be Peterson graph, then the strong domination number of H in context of vertex switching is 4.
Proof: Let H be a Peterson graph which is an undirected graph featuring 10 vertices and 15 edges. Let $\{p_1, p_2, p_3, \dots \dots, p_m\}$ be the vertices of H and let D be the dominating set of H. The labeling begins from the switched vertex, considered as p_1 and proceeds in any direction to label the remaining vertices. The vertex p_1 will be connected to all other vertices of the graph H except the three vertices which are not adjacent to each other. The dominating set D includes these three vertices along with the vertex p_1.
Hence, the Strong Domination Number of H in context of vertex switching is 4.

Example 2.8 The Strong Domination Number of Peterson graph in context of vertex switching is 4.

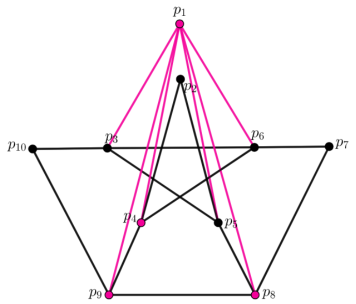


Fig 8: Peterson graph in context of vertex switching

III. Conclusion

In this paper, we have investigated the strong domination number of different families of graphs like barbell – graph, pan graph, sun – let graph and Petersen graph. The concept of domination number and strong domination number can be helpful in computer and networking, Biological Sciences and engineering. In addition, one can investigate the change in different kinds of domination number for different families of graphs and also using reconstruction of the graphs.

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