Discriminants of Brauer Algebra

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Abstract: In this work we study how to compute the brauer algebra discriminant and also define a matrix \( Z_{m,k}(x) \).

INTRODUCTION:
In the beginning of 20th century invariant theorists began to study the commuting algebras of the tensor powers of defining representations for the classical groups \( G = \text{GL}(n,C), \text{SL}(n,C), \text{O}(n,C), \text{So}(n,C) \) and \( \text{Sp}(2m,C) \).

These algebras may be defined as follows. Let \( G \) be a classical group. Let \( V \) be its defining representation, and let \( T^fV \) be the \( f \)-th tensor power of \( V \). (i.e.,) \( T^fV = V \otimes V \otimes \ldots \otimes V \). The group action of \( G \) on \( V \) lifts to the diagonal action of \( G \) on \( T^fV \) defined by \( g.(V_1 \otimes V_2 \otimes \ldots \otimes V_f) = (gV_1) \otimes (gV_2) \otimes \ldots \otimes (gV_f) \).

Define the commuting algebra \( \text{End}_{gV} \left(T^fV\right) \) of this action to be the algebra of all linear transformations of \( T^fV \) which commute with this action of \( G \). In the case of \( G = \text{GL}(n,C) \), Schur showed that there is a surjective algebra homomorphism from \( C_S^f \) onto \( \text{End}_{gV} \left(T^fV\right) \). In the case of \( G = \text{O}(n,C) \) and \( \text{Sp}(2m,C) \), Richard Brauer defined two algebras \( A^f \) and \( B^f \), where \( f \) is a positive integer and \( x \) is a real indeterminate. The surjective homomorphism then Richard Brauer failed to give the complete explanation of these kernels, Phil Hanlon and David Wales began to study the structure of the algebras \( A^f \) and \( B^f \) where \( x \) is an arbitrary real. The algebras \( A^f \) and \( B^f \) are isomorphic to each other. So it was only necessary to study the algebra \( A^f \). The authors were able to describe the radicals of \( A^f \) and the matrix ring decomposition of \( A^f / \text{Rad}(A^f) \). Later this problem was reduced to the problem of computing the ranks of certain combinatorially defined matrices \( Z_{m,k}(x) \).

1. Defining the matrix \( Z_{m,k}(x) \):
The computational problem will be to compute the rank of certain combinatorially defined matrices \( Z_{m,k}(x) \) for every complex number \( x \). The determinant of \( Z_{m,k}(x) \) is known to be nonzero as a polynomial in \( x \). So the rank of \( Z_{m,k}(x) \) is completely determined except at a finite number of values of \( x \). The finite values of \( x \) are those \( x \) that are the roots of \( \text{det}(Z_{m,k}(x)) \). So the computational problem breaks into two parts:
1. Compute the roots of \( \text{det}(Z_{m,k}(x)) \).
2. For each root \( r \), compute the rank of \( Z_{m,k}(r) \).

1.1. Definition:
Let \( m \) and \( k \) be nonnegative integers. An \((m,k)\) partial 1-factor is a graph with \( m+2k \) points and \( k \) lines which satisfies:
1. Every point has degree 0 or 1.
2. The \( m \) points of degree 0 are labeled with the numbers 1, 2... \( m \).

For example,

\[
\begin{array}{cccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

is a (6, 2) factor.

1.2. Notation:
Here we use ‘\( f \)’ to denote \( m+2k \), and lower case Greek letters \( \delta, \delta_1, \delta_2, \ldots \) to denote partial 1-factors. \( B_{m,k} \) denote the set of all \((m, k)\) partial 1-factors. Let \( V_{m,k} \) be the complex vector space with basis \( B_{m,k} \). The points of degree 0 are called the free points of \( \delta \).

1.3. Definition:
Let \( \delta_1 \) and \( \delta_2 \) be elements of \( B_{m,k} \). The union of \( \delta_1 \) and \( \delta_2 \) is a graph consisting of some number \( \gamma(\delta_1,\delta_2) \) of cycles together with \( m \) paths \( P_u \). If \( u \) is an endpoint of \( P_u \), then \( u \) is a free point of either \( \delta_1 \) or \( \delta_2 \). Hence, the end points of each path are labeled. We say \( \delta_1 \) and \( \delta_2 \) are consistent if each path of \( \delta_1 \cup \delta_2 \) has the property that its endpoints have the same label. Otherwise, \( \delta_1 \) and \( \delta_2 \) are inconsistent.

1.4. Definition:
Let \( m \) and \( k \) be nonnegative integers. Define a matrix \( Z_{m,k}(x) \) with rows and columns indexed by \( B_{m,k} \). For \( \delta_1, \delta_2 \in B_{m,k} \), let the \((\delta_1,\delta_2)\)-th entry of \( Z_{m,k}(x) \) be defined by

\[
Z_{m,k}(x)_{\delta_1,\delta_2} = \begin{cases} 
  x^{\delta_1(x) - \delta_2(x)} & \text{if } \delta_1 \text{ and } \delta_2 \text{ are consistent} \\
  0 & \text{if } \delta_1 \text{ and } \delta_2 \text{ are inconsistent}.
\end{cases}
\]

1.5. Note:
Each diagonal entry of \( Z_{m,k}(x) \) is \( x^k \) and that every off-diagonal entry is either 0 or \( x^e \) with \( e<k \). So the determinant of \( Z_{m,k}(x) \) is a nonzero polynomial in \( x \) of degree \( |B_{m,k}| \).

1.6. Example:
Let \( f=4 \) and \( m=2 \). In this case, the matrix \( Z_{m,k}(x) \) is 12 x 12.
An ordered basis for $B_{m,k}$ is given below:

Thus, the matrix $Z_{m,k}(x)$ with respect to this is given by

$$(2+2*1)! / 2^1 1! = 4! / 2 = 4*3/2 = 6$$

$\Phi$

Let $G$ be a finite group with irreducible representation

**Step 2:**

$Z_i$ can be computed as follows:

**Step 3:**

Let $v_i^{(0)}$ be the subspace of $V$ spanned by $\Phi(e_i^{(0)}) v_i ... \Phi(e_{m_i}^{(0)}) v_{m_i}$. The space $v_i^{(0)}$ is $Z$-invariant and $Z_i$ is the restriction of $Z$ to $v_i^{(0)}$.

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### 2. Computing the Brauer Algebra Discriminants

**2.1. Lemma:**

Let $\mu$ and $\lambda$ be partitions of $m$ and $f$ respectively, and let $m(\mu, \lambda)$ denote the multiplicity of $\Phi_\mu \otimes \Phi_\lambda$ in $V_{m,k}$. Then

$$m(\mu, \lambda) = \sum_{\eta \vdash 2k \quad \eta \text{ even}} g_{\mu \eta} \eta \vdash m$$

**Proof:**

Let $G$ be $S_f \times S_m$

Let $H$ be the subgroup $(S_{2k} \times S_m) \times S_m$, and let $S$ be the subgroup of $H$ given by $S = (\pi, \sigma, \tau) : \pi \in B_{2k}, \sigma \in Sym(m)$

Here, $B_{2k}$ denotes the hyperoctahedral group of $k \times k$ signed permutation matrices, which is considered to be a subgroup of $S_{2k}$.

$G$ acts as a transitive permutation group on the set $B_{m,k}$. So the action of $G$ on $V_{m,k}$ is the induction of the trivial character $\varepsilon$ from the stabilizer of any $\Delta \in B_{m,k}$ to $G$.

Choose $H$ to have the stabilizer of $\Delta_0$ is $S$.

Using a theorem of Littlewood and some well-known facts about the structure of group algebras, we have

$$\text{ind}_H^G (\varepsilon) = \sum_{\eta \vdash 2k \quad \eta \vdash m} g_{\mu \eta} \Phi_\mu \otimes \Phi_\eta$$

By the Littlewood–Richardson rule we have for each $\eta \mu$

$$\text{ind}_H^G (\Phi_\eta \otimes \Phi_\mu) = \sum_{\lambda \vdash \mu + \eta} \Delta^{\lambda}_\mu$$

Hence the theorem is proved.

Now, fix partitions $\mu \vdash m$ and $\lambda \vdash f$. If $\mu$ is not contained in $\lambda$, then $g_{\mu \mu} = 0$ for all $\eta$, So $m(\mu, \lambda) = 0$.

Therefore, we may assume that $\mu \subseteq \lambda$.

For that, we will have to identify a particular idempotent ‘$e$’ in the group algebra of $G$ corresponding to the irreducible representation $\Phi_\mu \otimes \Phi_\mu$. To obtain the idempotent ‘$e$’, first let $S_0$ be the minimal standard young tableau of shape $\mu$.

Therefore, $S_0 = (\mu_1+1) \quad (\mu_1+2) \quad ... \quad (\mu_1+\mu_2)$

Next, let $t_0$ be the standard young tableau of shape $\lambda$ which agrees with $S_0$ on the intersection of $\lambda$ and $\mu$ and which has the minimal filling of $[\lambda / \mu]$ with $m+1,...,f$.

Therefore,
Then \( e \) defined by \( e = e_o x e_o \) is \( e \) is an idempotent in the group algebra of \( G \) corresponding to the irreducible \( \phi_o \otimes \phi \).

### 2.2. Definition:

The pair \((\lambda, \mu)\) is \( \mu \) - external if \([\lambda / \mu]\) has no pair of squares in the same row or the same column. If \((\lambda, \mu) = \mu \) - extremal, then the tableau \( t_0 \) looks like:

\[
\begin{array}{cccccc}
1 & 2 & \ldots & \mu_1 \\
(m+1) & (m+\lambda_2 - \mu_1) \\
t_0 = (\mu_1 + 1) & (\mu_2 + 2) & \ldots & (\mu_1 + \mu_2) & (m + \lambda_2 - \mu_1 + 1) \\
(\mu_1 + \ldots, \mu_1 + 1) & \ldots & m \\
(f - \lambda_3 + 1) & \ldots & f
\end{array}
\]

Any lattice permutation of length \( 2k \) and shape \( \eta \) constitutes a littlewood – Richardson filling of \([\lambda / \mu]\). So for all \( \eta \) we have,

\[
m(\mu, \lambda) = \sum_{\eta \leq \lambda} f_{\eta} = \sum_{\eta \leq \lambda} f_{\eta} = 1.3 \ldots (2k - 1)
\]

Hence the multiplicity \( m(\mu, \lambda) \) equals the number of 1-factors on \( 2k \) points.

### 2.3. Note:

For any pair \((\lambda, \mu)\) we have \( g_{}\leq \lambda \leq \mu \leq \eta \).

Equality is achieved if and only if \((\lambda, \mu)\) is \( \mu \) - extremal.

### 2.4. Definition:

Let \( \Delta \) be a 1- factor with \( 2k \) points. Define the \( (m, k) \) partial 1- factor \( V_\Delta \) as follows:

1. \( V_\Delta \) has free points \( 1,2, \ldots, m \). The free point \( 1 \) has label \( j \).
2. For every edge \((u, v)\) of \( \Delta \) we have the edge \((m+u, m+v)\) of \( V_\Delta \).

The following lemma will not only show that the \( e V_\Delta \) linearly independent, it will also greatly streamline our computation. This result is difficult to prove in the case of general pairs \((\lambda, \mu)\).

### 2.5. Lemma:

Let \((\lambda, \mu)\) be \( \mu \) - extremal and define \( t_o, s_0 \) as above. Let \( \gamma, \sigma, \gamma' \) and \( e \) be in \( C_{\bar{o}} \), \( R_{\bar{o}} \), \( C_{\bar{o}} \) and \( R_{\bar{o}} \), respectively (so, sgn \( (\gamma) \) sgn \( (\gamma') \) sgn \( (\sigma) \) sgn \( (\gamma') \) is one of the terms occurring in the idempotent \( e = e_o x e_o \)). Suppose that:

\[(\gamma, \sigma, \gamma') V_\Delta = V_\Delta, \text{where} \Delta \text{\ and} \Delta_i \text{\ are 1-factors. Then}
\]

1. \( \Delta = \Delta_i \).
2. \( \gamma \) and \( \sigma \) both fix \( t_o / s_0 \) point wise.
3. \( \gamma \) restricted to \( s_0 \) equals \( \gamma' \) and \( \sigma \) restricted to \( s_0 \) equals \( \sigma' \).

In particular, \( e V_\Delta : \Delta \) is a 1- factor with \( 2k \) points is a basis for \( e V_{m,k} \).

### Proof:

Let \( \gamma' \) acts on an \( m,k \) Partial 1-factor by changing the labels on the free points by \((\gamma', \sigma') \).

Since the free points of both \( \Delta \) and \( \Delta_i \), are \( 1,2, \ldots, m \), it follow that \( \gamma' \)\( \sigma' \) preserves the sets \{ \( 1,2, \ldots, m \} \) and \{ \( m+1, \ldots, f \) \}.

In \( t_o \) each square \( m+u(\mu = 1,2, \ldots, 2k) \) is at the right-hand end of the row containing it and at the bottom of the column containing it.

So \( \sigma \) moves the point \( m+u \) weakly to the left. Since the image of \( m+u \) under \( \gamma' \) is in the set \{ \( m+1, \ldots, f \) \}, the permutation \( \gamma \) must then move \( \sigma(m+u) \) down to the bottom of the column it occupies.

Thus, \( \gamma' \sigma \) acts on \( m+u \) point wise. Now, consider \( \sigma \) x \( \gamma' \sigma' \) on \( s_0 \). The point \( j \) is moved by \((\gamma, \gamma' \sigma') \) to \((\gamma' \sigma') \) and its label is changed to \((\gamma' \sigma') \) j for all \( j \).

Thus, \( \gamma = \gamma' \sigma' \), so \( \gamma' \) is \( \gamma' \) and \( \sigma = \sigma' \) where these last three equalities refer to \( \gamma, \gamma' \) and \( \sigma \) restricted to the points of \( s_0 \).

Suppose \( \Sigma_{\gamma(\sigma)} V_\Delta = 0 \). Then \( \Sigma_{\gamma(\sigma)} V_\Delta = 0 \), and so all \( a_x = 0 \), as the \( V_\Delta \) are linearly independent.

Thus the set \{ \( e V_\Delta \) \} is a basis.

### 2.6. Definition:

Let \((a_i, b_i)i(1,2, \ldots, 2k)\) be the co-ordinates of the squares of \([\lambda / \mu]\).

For each \( i \) define sets \( C_i \subseteq C_{\bar{o}} \) and \( R_i \subseteq R_{\bar{o}} \) as follows:

1. \( C_i \) contains the identity permutation as well as the \((a_i,1)\) involutions \( \gamma_i \) which exchange the elements of \( t_o \) in squares \((a_i,1) \) and \((j,1)\).
2. \( R_i \) contains the identity permutation as well as the \((b_i,1)\) involutions \( \gamma_i \) which exchange the elements of \( t_o \) in squares \((a_i,1) \) and \((j,1)\).

Let \( C \) be the set of all products \( \gamma_{(i)} \ldots \gamma_{(2k)} \), where \( \gamma_{(i)} \) \( C_{\bar{o}} \), and let \( R \) be the set of all products \( \sigma_{(i)} \ldots \sigma_{(2k)} \), where \( \sigma_{(2k)} \) \( C_{\bar{o}} \) and \( R_{\bar{o}} \) are subsets of \( C_{\bar{o}} \) and \( R_{\bar{o}} \) of sizes \( m \times m \).

Hanlon and Wales algorithm:

Let \( G \) be a finite group with irreducible representation \( \Phi_i, \Phi_i \) \( \Phi \) is a representation of \( G \) on a complex vector space \( V \) which decomposes into irreducible as \( e = \Sigma_{i} \Phi_i \).

Let \( Z \) be a linear transformation of \( V \) which commutes with the action of \( G \). Then \( Z \) is similar to a matrix which is a direct sum over \( i \) of matrices \( Z_i \), where \( Z_i \) is an \( m \times m \) matrix repeated in the direct sum deg(\( \Phi_i \)) times. Moreover, \( Z \) can be computed as follows:

1. Choose a complete set of Primitive orthogonal idempotents \( e_i \), \( 1 \leq u \leq \leq 5 \leq \deg(\Phi_i) \) in the group algebra \( CG \).
2. Find \( m_i \) vectors \( v_{i,1} \ldots v_{i,m_i} \) such that \( \Phi(e_i(v_{i,1} \ldots v_{i,m_i}) V \) is linearly independent.

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Step 3:-
Let $v_i(0)$ be the subspace of $V$ spanned by $\Phi(e_i(0)) v_1 \ldots \Phi(e_i(0)) V _{m0}$. The space $v_i(0)$ is $Z$-invariant and $Z_i$ is the restriction of $Z$ to $v_i(0)$.

The above algorithm will compute the $V_{A2}$, $V_{A1}$ entry in $Z_{m,i}(x)$ as a sum of terms of the form $\tau = (\gamma, \sigma') V_{A2}$ $V_{A1}$, where $\gamma \in \Gamma, \sigma' \in R_{m0}$. For a fixed pair $(r, \sigma) \in X \times R_{m0}$ there is almost one pair $(r', \sigma') \in X \times R_{m0}$ for which $r$ is nonzero.

We next write down a method for computing $\pi = Y \sigma$ given $(Y \sigma)$ $V_{A2}$ and $V_{A1}$. In the description below we will assume that the in put is $\delta_1 = Y \sigma V_{A2}$ and $\delta_2 = V_{A1}$.

2.7. Definition:
Let $\delta_1$ and $\delta_2$ be (m, k) partial 1-factors. Define an element $\pi (\delta_1, \delta_2)$ in the group algebra CS $m_0$ according to the following algorithm.

For each $i$ in the set $\{1, 2, ..., m\}$ find the unique path in $\delta_1 \nu_2$ which begins at the free point of $\delta_1$ labelled $i$ and ends at some other free point $y$. If $y$ is a free point of $\delta_1$, then $\pi(\delta_1, \delta_2)$ = 0 and algorithm stops. Otherwise, $y$ is a free point of $\delta_2$. Let $\pi (\delta_1, \delta_2)(i)$ be the label on $y$.

When this algorithm finishes, we will have either, $\pi (\delta_1, \delta_2) = 0$ or else $\pi (\delta_1, \delta_2) \in S_m$. For $\delta_1, \delta_2$ both (m, k) partial 1-factors and $r$ a standard young tableau of size $m$, define $\Gamma_r (\delta_1, \delta_2)$ by

$$\Gamma_r (\delta_1, \delta_2) = \begin{cases} 0 & \text{if } \pi (\delta_1, \delta_2) = 0 \text{ the coefficient of } \pi (\delta_1, \delta_2) \text{ in the young symmetrizer } e^c, \\
\text{if } \pi (\delta_1, \delta_2) \in S_m & \end{cases}$$

2.8. Lemma:
Let $Y = Y^{(1)} \ldots Y^{(2k)}$ be in C and $\sigma = \sigma^{(1)} \ldots \sigma^{(2k)}$ are not identity. Then $\Gamma_{S_2} (Y \sigma V_{A2}, V_{A1}) = 0$.

Proof:-
Fix $i$ such that $Y^{(i)}(u, b_i)$ with $u < a_i$ and $\sigma^{(i)}(a_i, v)$ with $V < b_i$.

Let $a$ and $b$ be the labels in the squares (a, b) and (a, v) of $Y$. The row permutation $\sigma$ moves the label $b$ to the corner square (a, b).

Then the column permutation $Y$ moves the label to the square (a, b).

So in $Y \sigma V_{A2} u V_{A1}$, the path beginning at the free point labeled $b$ in $Y \sigma V_{A2}$ has length 0 and ends at the same point of $V_{A1}$ which is a free point labeled $a$.

So, $\pi (Y \sigma V_{A2}, V_{A1})(b) = a$.

But the corner Square (a, b) does not exist in $S_m$, so $\pi (Y \sigma V_{A2}, V_{A1})$ moves $b$ from position (a, v) to (u, b) where $b_i > a_b$.

It is easy to see that such a permutation cannot be written in the form $Y \sigma$ where $Y \in C \in R_{m0}$ and $\sigma \in S_m$.

So, $\Gamma_{S_2} (Y \sigma V_{A2}, V_{A1}) = 0$.

This couples the proof.

2.9. Note:
Let $S$ denote the set of Pairs $(Y \sigma)$ with $Y = Y^{(1)} \ldots Y^{(2k)} \in C$ and $\sigma = \sigma^{(1)} \ldots \sigma^{(2k)} \in R$ Such that $\sigma^{(1)}$ is the identity wherever $Y^{(1)}$ is not the identity.

The size of $S$ is $|S| = \pi(ai + bi - 1)$.

2.10. Theorem:
The following algorithm computes the $\Delta_i \Delta_j$ entry in $Z_{m,i}(x)$.

Algorithm for each pair $(\gamma, \sigma) \in \Gamma$

1. Compute $\Gamma_{S_0} (\gamma \sigma V_{A2}, V_{A1})$.

2. Compute the number of cycles $N$ in $\gamma \sigma V_{A2} UV_{A1}$.

3. Add $\text{Sgn}(\gamma) \Gamma_{S_0} (\gamma \sigma V_{A2}, V_{A1}) x^N$ to the current value of $Z_{m,i}(x)$.

Before proving this algorithm consider the case $\lambda = (6,5,4,3,2,1)$ and $\mu = (5,4,3,2,1)$.

The size of original matrix $Z_{m,i}(x)$ is a whopping $(21)!/48$.

The submatrix $Z_{m,i}(x)$ is $15 \times 15$. The six squares of $[\gamma / \mu]$ have co-ordinates $(1,6),(2,5),(3,4),(4,3),(5,2)$ and $(6,1)$, so the size of $S$ is $6^6$.

Thus each entry of $Z_{m,i}(x)$ is computed with $6^6$ passes through the main loop of the algorithm in above theorem.

In practice, this matrix $Z_{m,i}(x)$ was computed in about one hour of CPU time on a CRAY-2.

In general, we must perform the main loop in the above algorithm $\pi(a_i + b_i - 1)$ times. This main loop is carried out in $O(\Gamma_{S_0} (\gamma \sigma V_{A2}, V_{A1}))$ steps.

So, the above Theorem gives a method to compute each entry of $Z_{m,i}(x)$ in $0 \pi(a_i + b_i - 1) (1 + \Gamma_{S_0} (\gamma \sigma V_{A2}, V_{A1}))$ steps.

Proof:
Let $e^c = e^c_{o \sigma}$ be the young symmetrizer indexed by $s_0$. According to Theorem, Let $G$ be a finite group with irreducible representation $\Phi_1, \Phi_2$. Let $\Phi$ be a representation of $G$ on a complex vector space $V$ which decomposes into irreducible as $\Phi = \sum_{i=1}^{c} \Phi_i$. Let $Z$ be a linear transformation of $V$ which commutes with the action of $G$. Then $Z$ is similar to a matrix which is a direct sum over $i$ of matrices $Z_i$, where $Z_i$ is an $m_i \times m_i$ matrix repeated in the direct sum $\text{deg} (\Phi_i)$ times. Moreover, $Z_i$ can be computed as follows:

Step 1: Choose a complete set of Primitive orthogonal idempotents $e_i(0), 1 \leq i \leq c, 1 \leq i \leq \text{deg} (\Phi_i)$ in the group algebra $CG$.

Step 2: Find $m_i$ vectors $v_1 \ldots v_{m_i} e V$ such that $\Phi(e_i(0)) v_{m_i}$ are linearly independent.

Step 3:-
Let $v_i(0)$ be the subspace of $V$ spanned by $\Phi(e_i(0)) v_1 \ldots \Phi(e_i(0)) v_{m_i}$. The space $v_i(0)$ is $Z$-invariant and $Z_i$ is the restriction of $Z$ to $v_i(0)$.

The matrix $Z_{m,i}(x)$ preserves the subspace $\{v_{m,i} \Delta \}$, which is a 1-factor of size $2k_i$. By Lemma, Let $(\lambda, \mu)$ be $\mu$-extremal and define $t_{o0}, s_0$ as above. Let $\gamma, \sigma, \gamma'$ and $\sigma'$ be in $C_{so}$ $R_{so}$ and $R_{so}$, respectively (so, $\text{sgn} (\gamma) = \text{sgn} (\gamma') = (\gamma, \sigma, \gamma')$ is one of the terms occurring in the idempotent $e = e_{so} x e_{so}$). Suppose that $\gamma \sigma', \gamma' \sigma \in \Gamma_{S_0} (\gamma \sigma V_{A2}, V_{A1})$, where $\Delta = \Delta_1$ and $\Delta_1$ are 1-factors. Then $\Delta = \Delta_1$.

5. $\gamma$ and $\sigma$ both fix $t_{o0}$ and $s_0$ point wise.

6. $\gamma$ restricted to $s_0$ equals $\gamma'$ and $\sigma$ restricted to $s_0$ equals $\sigma'$.

In particular, $\{v_{m,i} \Delta : \Delta = \text{a 1-factor with 2k points}\}$ is a basis for $e_{m,i}$.

The co-efficient of $v_{m,i}$ in $e_{m,i} v_j$ is 0 for $i \neq j$ and is $|R_{o0} \mid C_{so}$ for $i = j$. 
so the $i,j$ entry of $Z_{\alpha,\mu}(x)$ is $(1/ \left| R_{\alpha} \cap C_{\mu} \right| \left| C_{\alpha} \cap C_{\mu} \right|)$ times the coefficient of $v_{\Delta i}$ in $Z_{\alpha,\mu}(x)(ev_{\Delta j})$. Thus

$$\left( Z_{\alpha,\mu}(x) \right)_{\Delta i,\Delta j} = \frac{1}{\left| R_{\alpha} \cap C_{\mu} \right| \left| C_{\alpha} \cap C_{\mu} \right|} (ev_{\Delta j}, v_{\Delta i})$$

$$= \frac{1}{\left| R_{\alpha} \cap C_{\mu} \right| \left| C_{\alpha} \cap C_{\mu} \right|} \sum_{\gamma \in C_{\alpha}} \text{Sgn}(\gamma) \sum_{\sigma \in R_{\alpha}}$$

$$= \sum_{\gamma \in C_{\alpha}} \text{Sgn}(\gamma) \int_{S_0} (\gamma \sigma V_{\Delta j} V_{\Delta i}) (\gamma \sigma V_{\Delta j}, V_{\Delta i}) .$$

This equality follows from the definition $\int_{S_0}$ we have $\left( Z_{\alpha,\mu}(x) \right)_{\Delta i,\Delta j} = \sum_{(\gamma, \sigma) \in S} \text{Sgn}(\gamma) \int_{S_0} (\gamma \sigma V_{\Delta j} V_{\Delta i}) (\gamma \sigma V_{\Delta j}, V_{\Delta i}) .$

This completes the proof.

**CONCLUSION:**

I have tried to give a brief sketch of some of the main ideas underlying the dynamically growing field of Brauer Centralizer Algebra. It is the natural Convergence of ideas from many areas of mathematics such as algebra, combinatorics, with those from computers science, such as algorithms, data structures. I feel confident that the current trend of studying Brauer Algebra will continue to suggest new classes of problems which are can continue for further enrichment of his knowledge.

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