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Sensitive Dependence on Initial Conditions, and Chaotic Group Actions

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***Abstract:* - If there is a number $\varepsilon > 0$ such that for any open set u we can find such that $g \in g$ diameter greater than ε is a constant battle $U\, g$. A compact metric space is a sensitive dependence on initial conditions of group g we proved that if a compact metric space as a countable transitive g −system or at least a measure of probability Full support, protection, and serves or initial conditions on equicontinuous sensitive dependency. Periodicity, assuming we are the same conclusions without count ability. these theorems Glaser and extension of a theorem invertible case Weiss we prove that the system generated initial conditions sensitive dependency when a finitely-solvable Group serves as transitive and dense set of points less than fixed wheel sub actions. In addition we are non-compact non-transitive monotheism groups and, at least, almost equicontinuous, recurrent G-examples of actions that show how to build.**

Keywords: Sensitive dependence on initial condition; a probability measure; system minimal & equicontinuous; ergodicity; non-compact monothetic groups and transitive, non minimal.

1. INTRODUCTION:

A chaotic group action as a generalization of chaotic dynamical systems they showed that a group G acts chaotically on a compact Hausdorff space if and only if G is residually finite. They constructed chaotic action of the infinite cyclic group on the 2-disk and observed that the Special linear group $SLn\,(Z)$ acts chaotically the n-dimensional torus. In the present paper, we construct many chaotic group actions on (even-dimensional) spheres and tori using the examples of Cairns et al. as building blocks. More precisely we show, among other things, that many nightly generated infinite abelian groups act chaotically on even-dimensional spheres; (ii) any finite index subgroup of $SLn(Z), n > 2$, acts chaotically on the nk-dimensional torus for any $k \geq 1$. We also consider chaotic group actions on (connected) open manifolds where each element of the group is a compactly supported self homeomorphism. For a (self) - homeomorphism f of space X, $supp\,(f)$, the support of f, is defined to be the set $\{(x)\; 2\; X \mid f(x) \;\neq (x)$. We say that a homeomorphism f of X is compactly supported if $supp\,(f)$ is relatively compact, i.e., the closure of $supp\,(f)$ is compact. A basic result is that any compactly supported homeomorphism of a connected open manifold is of infinite order. The requirement that the group act chaotically (and effectively) via compactly supported homeomorphisms, leads to many interesting results of group theoretic nature. The results we obtain are by no means exhaustive. However, we do not know of a single chaotic group action via compactly supported homeomorphism on any open manifold. We conjecture that no such action can possibly exist on the Euclidean space $R^n, n \geq 2$. The strongest evidence for this conjecture is that it is true when one restricts attention to such classes of groups as solvable groups, groups with non-trivial center, groups which decompose as a direct product with one of the factors being finitely generated. However a complete resolution of the conjecture has eluded us. (It is not hard to show that no group can act chaotically on any 1-dimensional manifold, the case of the circle having been covered in [2].) Most of our results regarding compactly supported chaotic actions on open manifolds are valid for any noncompact, locally compact space and are treated in that generality. For the convenience of the reader, we reproduce below the definition of a chaotic action.

2. CHAOTIC ON OPEN MANIFOLDS:

Let X is a non-compact, locally compact, locally connected Hausdorff space. For a homeomorphism f of X, the support supp (f) of f is defined to be the set $\{x \in X \mid F(x) \neq x\}$ Denote by $G_0(X)$, the subgroup of those homeomorphisms f of X which are compactly supported, i.e., the closure of $supp\,(f)$ is a compact subset of X. In this Section, we obtain some general results concerning chaotic group actions on X. The most interesting example of such a space is an open manifold. Their importance justices our choice of the title for this section we begin with some elementary observations. A topological dynamical system is a compact metric space is a continuous operation by a topological group g will mean (X, d), i.e., a continuous map $\pi\colon g \times x \to x\colon (g, x)$ with $x \to g$. The $g\,(h.x) = gh$ and $e.x = x$, where e is the identity element of g we have a system that pair (X, g) denote by $t.\colon X \to X$ is a homeomorphism, topological dynamical system, we also have a (X, T), we can call the clear operation z we have assumed that T invertible theorems not to mention occasionally are referring to when this is the case it will be obvious. Dynamical systems to understand the following property is the primary goal of this paper.

Definition 2.1:

A topological dynamical system (X, G) is sensitive to the initial conditions is called a dependency (or is said to be sensitive) if there's some of $x \in X$ such that all $x > 0$ and every open neighborhood U of x, $y \in g$ such that you g and d $(g.g.x, y) > 0$ Is that one need not (overtly) mention points worth noting initial conditions; Definition of sensitive dependence on the following equivalent definition: open all $u, \varepsilon > 0\; supg \in G$ there exists set diam $(g.U)\; \varepsilon > 0$.

Definition 2.2:
A point $x \in X$ a is equicontinuous point, all $\varepsilon > 0$ there exists if $y \in x$ such that any $\delta > 0$, $d(x,y) < \delta$ implies $d(g.g.x,y) < \varepsilon \in g$ all g-A point that is not a sensitive point will be equicontinuous point (X,G) a system is said to be equicontinuous if all map sets. $\{X \in x: g \to g._g\}$ (X,G) equicontinuous, equicontinuous, and then $\varepsilon > 0$ and each point every point so it's easy to see every point equicontinuous. On the contrary, that is equicontinuous families. there are $x > 0$ such that for all $g \in g$ is less than ε for diameter G. Box (x) Box (x) has a finite sub cover of balls you can choose to let a Lévesque number Δ to be covered again, if d $(x,y) \in U$ containing $x,u < \delta$ and $y.,d(g.g.x,y) \leq diam(g.U)$ it all $g < \varepsilon$ proves that everything is equicontinuous system if and only if. Now we x A new metric d (x, y) ∞ defined by $d = supg \in g\,(g.g.x,y)$. Consider the identity map $Id:(X,d) \to (x,d\,\infty)$. When g is a mooned, Id-1 is a contraction and therefore continuous. If (X,g) is equicontinuous, ID is a homeomorphism is easy to see, if $(X,d\,\infty)$ compact, Haus dorff compact ID domain and range and then a homeomorphism is a fine crack point ID. Logically speaking is a sensitive issue, emphasized that a system focused on initial conditions sensitive dependency that every issue is stronger than the sensitive (they order in which quantifiers). [1]

Proposition2.3:
A transitive system is sensitive at each point if and only if it has sensitive dependence on initial conditions.

Definition2.4:
If it is a dense set of points is an equicontinuous system almost equicontinuous. it systems for a single transitive maps, sensitive, or set of points at all equicontinuous transitive points is the same as that of [3] estimated 2 thus it usually Aulander-Yorke virodhabhAsa theorem is known as well as non-invertible case. Following standard definition are discussed in much more detail elsewhere. For example, [11] and [6] good reference from now on we take to be a group of us (X,g) says that you and V open present If $g \in g$ is a topologically transitive (transitive or for short) that $U \cap g.V = \emptyset$. our settings, it is set to $X \in x_0$ points of equal to the existence of a dense, such as G. x_0 is a System x in dense at least has a dense orbit every point is called a point at least if your class has a minimal system off Will be called when the group is discrete, if acting is a point $p \in X$ is a finite view periodic point. Periodic points are easily to be minimized. saying that x is a minimal point that every open neighborhood of x,r: is equal to the required $r = \{g \in g.: X \in U\}$ left g. are synthetic; That is, there is a finite subset F of g such that $g = FR$ in a group, any defined form SS-1 is an infinite set of $= \{s,t \in S\,main\,s.t\,1:\}$ is called a Δ-set if it intersects a set Δ set is called a set that is symmetrical (that is, identical to its inverse) is always looking for synthetic Set $L \subset g$ is symmetric but not synthetic. Let $g_0 = 1\,G$ and g_0, we have different elements gn-1 is already chosen with this value that gig-1 j ∈ j/L when I _ not since L = that synthetic. $\{gng - 10,\dots,n - 1 - 1\,gng\}$ l recalls we can choose gn we also know since L symmetric $\{g0g - 1n,\dots,Gn - 1g - 1n\}\,l$ Inductively, we see $that\,S \cap L\,SS - 1\,n = \{gn\}\,where = \emptyset$; That is, $L\,\Delta\,\sigma$-algebra b is North _ on a probability $measure\,\mu = B(X) = \mu\,If\,\mu(A)$ real estate $(g - 1.A\,B\,for\,all\,a \in), g \in g$ North measures to be assumed all measures. measures μ measure zero argotic if one or any real estate is said to be founded on literature, the result is some extra sensitivity hypothesis under a common theme, it almost has to be equicontinuous system gives a contradiction In addition to satisfying the hypothesis: any system either sensitive or equicontinuous. Topological hypotheses are popular (see [4], [6] and [7]). Measure theoretical hypotheses also appear (see [7]). The next two theorems are especially good example. [3]

Theorem2.5:
(Glaser and Weiss [7]) If (X,T) is a transitive topological system equipped with an invariant probability measure μ of full support, then either (X,T) has sensitive dependence on initial conditions or it is minimal and equicontinuous.

Theorem 2.6:
(Akin, Aulander, and Berg [1]) Let (X,T) be a (possibly noninvertible) transitive system. If the set of all minimal points is dense, then either the system has sensitive dependence on initial conditions or X is a minimal equicontinuous system. Both of these theorems are generalizations of their predecessor in [4]:

3. ALMOST EQUICONTINUITY AND TRANSITIVITY
In this section we have many transitive, equicontinuous systems which are not less than the very existence of equicontinuous. We [2] theorems 4.2 and 4.6 analogues proved virtually equicontinuous, but not to eliminate the possibility of equicontinuous systems additional hypotheses discussed in paradox type theorems use this type of example that exists without the knowledge The strength of this kind of dubious, theorem.

Lemma3.1:
Let (X,G) be a topological dynamical system with an equicontinuous point x. If $y \in X, Gn \in G\,and\,gn.y \to x$, then y is also equicontinuous and has the Same orbit closure as x.

Proof.
Given $\varepsilon > 0$ there is a neighborhood $U\,of\,x$ such that diam $(g.U) < \varepsilon$ for all $g \in G$. Fix h such that $h.y \in U$. Then $V = h - 1.U$ is a neighborhood of y with $diam\,(g.V) < \varepsilon$ for all $g \in G$. Thus y is an equicontinuous point. Since $d(g.x, gh - 1.y) < \varepsilon$ for all $g \in G$, we see that the orbit of x is ε-dense in the orbit of y and the orbit of y is ε-dense in the orbit of x. Thus, taking closures of each orbit yields equal sets. _

Proposition3.2:
$Let\,(X,G)$ be a system with a transitive point x. The following are equivalent:
(1) (X,G) is almost equicontinuous.
(2) X is an equicontinuous point.
(3) For all $\varepsilon > 0$ there exists $\delta > 0$ such that for all $g,h \in G, d(h.x,x) < \delta$ implies $d(gh.x,g.x) < \varepsilon$.
(4) D and d∞ induce the same topology on the set of transitive points.

Proof.
Assume (2). Then any translate of x is an equicontinuous point, and (1) follows. Now we prove that (3) implies (2). Fix $\varepsilon > 0$ and choose $\delta > 0$ so that $d\,(h.x, x) < \delta$ implies $d\,(gh.x, g.x) < \varepsilon/2$ for all $g \in G$. Now fix $g \in G$ and suppose $d(x, y) < \delta/2$. Choose h to make $h.x$ close enough to y that $d\,(gh.x, g.y) < \varepsilon/2$ and $d(x, h.x) < \delta$. Then $d\,(g.x, g.y) \le d\,(g.x, gh.x) + d\,(gh.x, g.y) < \varepsilon/2 + \varepsilon/2$.
We will now prove that (1) implies (4) and (4) implies (3). For (1) implies (4), it suffices to show that a sequence x^n of transitive points converges under d to another transitive point x if and only if the same is true under d∞. One direction is obvious. For the other direction, suppose $x^n \to x$ under d. By assumption x is transitive and hence equicontinuous by Lemma 2.1. For any $\varepsilon > 0$, when n is sufficiently large, we have $d\,(g.xn, g.x) < \varepsilon$ for all $g \in G$; that is, $d\infty\,(xn, x) \le \varepsilon$. Therefore $d\infty\,(xn, x) \to 0$, as desired to see that (4) implies (3), suppose $HN \in G$ are such that $d\,(hn.x, x) \to 0$. Then $d\infty\,(hn.x, x) \to 0$ as well, whence (3). [8][6]

Proposition3.3:
Suppose that G acts transitively by isometrics on a possibly non-compact metric space X_0 and $\iota: X_0 \to X$ is a uniformly continuous metric G-compactification; i.e., (X, G) is a compact metric system and ι is a uniformly continuous, G-equivariant homeomorphism embedding of X_0 onto a dense subset of X. Then (X, G) is an almost equicontinuous, transitive system with $\iota(X_0)$ contained in the transitive points of X. Conversely, every almost equicontinuous, transitive system (X, G) arises in this way: X0 may be taken to be the set of transitive points equipped with the d∞ metric.

Proof. Since G acts transitively by isometrics on X_0, it acts minimally. Thus every Point $y \in X_0$ has orbit dense in X_0 and so $\iota(y) = x$ has orbit dense in X. We now must show that x is an equicontinuous point. Fix $\varepsilon > 0$, and let $x\_ := \iota\,(y\_)$. Using the continuity of $\iota - 1$ and uniform continuity of ι we can choose $\delta > 0$ such that if $d(x, x\_) < \delta$, then for all $g \in G, g.y$ and $g.y\_$ are sufficiently close that $d\,(\iota\,(g.y), \iota\,(g.y\_)) = d\,(g.x, g, x\_) < \varepsilon$. In other words, x is an equicontinuous point. For the converse, let (X, G) be an almost equicontinuous, transitive system and let X_0 be the set of all transitive points equipped with the $d\infty$ metric. Then the inclusion $\iota: X_0 \to X$ is a contraction and hence uniformly continuous. By Proposition 2.2, part (4), ι is a homeomorphism embedding. So, in fact, ι is a uniformly continuous G-compactification, as desired. According to Proposition 2.3, constructing almost equicontinuous systems is equivalent to constructing equicontinuous compactifications of transitive isometric G-actions. One simple way to construct an almost equicontinuous G-action is to let $X_0 = G$ with the metric $d\,(g, h) = 1$ if and only if $g\_ = h$. This is an invariant metric giving the discrete topology. One could then take the one-point compactification of G and extend the left multiplication action of G by fixing the point at infinity. The infinite point is sensitive and all other points are equicontinuous and transitive.
From a topological perspective, this example is not very interesting. Notice that, except for the point at infinity, none of the transitive points are recurrent; i.e., it is not true that for every transitive point x, every neighborhood U _ x, and every compact $K \subset G$ we can find $g \,/\!\in K$ with $g.x \in U$. If $G = Z$ we can construct more examples by taking X = X0 to be some compact monothetic group (a group with a dense cyclic subgroup). Such examples are well known and abundant. They occur precisely as Pontryagin duals of subgroups of the circle equipped with the discrete topology. Again, these examples are not very dynamically interesting because they are minimal and equicontinuous. If we take X_0 to be a non-compact monothetic group, then we can easily pick a metric on it with respect to which Z acts by isometrics. Then any uniform compactification $\iota: X_0 \to X$ gives a transitive almost equicontinuous system which is not minimal and equicontinuous. In fact, it is not hard to see that any transitive isometric Z-action on a complete metric space X is actually a monothetic group. This is explored in detail in [2]. [9]
Now we will show how to construct an example of a G-system which is transitive, almost equicontinuous, and recurrent, but not minimal and equicontinuous. If one analyzes the procedure, we exploit the existence of non-compact monothetic groups.
Some examples of such groups are known (see [10]) and any of them may be used in our construction. We will show a different method for constructing such groups.
First we prove some obvious propositions which reduce the problem of defining isometric transitive G-actions to defining norms on G.

Definition3.4:
A symmetric norm on a group G is a function $g\,\_ \to g \in [0, \infty]$ such that $g = 0$ if and only if $g = 1, g = g - 1$, and $GH \le g + h$. Two norms · i, i = 1, 2, are uniformly equivalent if for all $\varepsilon > 0$ there is a δ > 0 such that for any i $and\ j, gi < \delta$ implies $gj < \varepsilon$.

Proposition2.5. Transitive, isometric, free G-actions on complete, pointed metric spaces are in one-to-one correspondence with symmetric norms on G.

Proof. First, suppose G acts transitively, isometrically, and freely on a complete pointed metric space (X, x). Define $g = d(x, g.x)$. Since the action is free, $x\,\_ = g.x$
Unless $g = 1$. Since the action is isometric, $g - 1 = d(x, g - 1.x) = d\,(g.x, x) = g$. Finally $GH = d(x, gh.x) \le d(x, g.x) + d\,(g.x, gh.x) = d(x, g.x) + d(x, h.x) = g + h$. Now suppose we have a symmetric norm · on G. Then we can define a left invariant metric ton G by $d\,(g, h) = g - 1h$. Let X be the completion of G with respect to this metric and choose $x = 1$ for the base point. Then G obviously acts on X transitively, isometrically, and freely. A point x is said to be recurrent if for every neighborhood $U\,\_x$ outside every compact subset of G we can find g such that $g.x \in U$. A G-action is said to be recurrent if every point is recurrent. Given a sequence $gn \in G$, we will say gn tends to infinity

and write $gn \rightarrow \infty$ if for any compact $K \subset G$, there are only finitely many n with $gn \in K$.[10]

4. SENSITIVE DEPENDENCE ON INITIAL CONDITIONS

$D\ (g.x,y) \leq h\ (g.g.x,y) + d\ (g.y,y) \in r\ we\ 2\delta 2 + \delta 2 < \delta 1 < d\ (x,g.x) \leq d\ (x,y) + (y,g.x)\ d\ 2\delta 2 < 3\delta 2 < \delta 1 + .(X,hg-1.x)\ d \leq d\ (x,x\ h.) + d\ h \in R$ take $(hg-1\ (g.x),hg-1.x)\ \varepsilon\ 3\delta 2 + \delta 3\ we <$ have proven that $< RR-1 \subseteq R\ (x,be(x))$. Since ε was arbitrary, it follows that any neighborhood of x times the set Δ _. In particular, r (x, $B\delta 3\ (x))\ \Delta \in g.r._(x,B\delta 3\ (x)),(x,g-1.x)\ d = (g-1.(G.x),g-1\ x)$ it follows that $R < \varepsilon\ (x,B\delta 3\ (x)) \cup R\ (x,B\delta 3\ (x)) - 1 \subseteq r\ (x,be\ (x))$. In other words, $r\ (x,be\ (x))$ is a symmetrical Δ _ and so ((as explained in the introduction) synthetic should be left). But asserting that x is at least equal to the classroom. X is transitive since, we conclude that (x,G) is minimized. Lemma 2.1 (X,g) equicontinuous. We now proceed to the proof of Theorem 1.9. Method is completely different: this offer depends on the following simple lemma 2 and first defined function $s\ (x) =$ in Support $\varepsilon > 0$ {diam $(g\ be\ (x)): g \in g\} = in\ \{d\ \infty - diam\ (U): x \in U$ shell}.

We notice that x is a sensitive issue if and only if s (x) x sensitivity constant calls s (x) > 0.

Lemma4.1:

The function s defined above is upper semi-continuous and hence measurable. If (X,G) admits an argotic measure μ of full support for which the set of all sensitive points has positive measure, then the system has sensitive dependence on initial conditions.

Proof.

Suppose U is a neighborhood of x such that $d\infty$-$diam\ (U) < s(x) + \varepsilon$. Then for any $y \in U, s(y) \leq d\infty - diam\ (U) < s(x) + \varepsilon$ Therefore s is upper semi-continuous. Measurability follows.

Since s is an invariant function it is constant μ-almost everywhere. Since it was assumed to take positive values on a set of positive measure, it must be equal to some $c > 0, \mu$-almost everywhere. Thus $s-1(c)$ is a dense set. Let U be any open subset of X. Then U contains an element of $s-1(c)$. By the definition of s we can find g ∈G such that $diam\ (g.U) > c/2$. So, (X,G) is sensitive. Proof of Theorem 1.9 Let (X,G) be an almost equicontinuous system and let μ be an argotic probability measure of full support. Then the set X0 of equicontinuity points is an invariant set and so must have measure 1 or 0. If it has measure 0, then almost every point is sensitive. By Lemma 3.2, (X,G) is sensitive. If X0 has measure 1, then by Proposition 2.3 we can think of μ as a Boreal measure on $(X0,d\infty)$.

If $(X0,d\infty)$ is not compact, then for some ε > 0 we can choose a sequence $x1,x2 \in X0$ with $d\infty(xi,xj) > \varepsilon\ when\ i\ \_= j$. Cover $X0$ by countably many balls of d∞-radius ε/4. One of them must have positive measure. Call it B and choose $Gn \in G$ such that $xn \in gn.B$. Since G acts on $(X0,d\infty)$ by isometrics, the balls gn.B is disjoint. This is ludicrous, since they are all of equal, positive measure. So, $(X0,d\infty)$ must be compact. Continuity of $(X0,d\infty) \rightarrow (X,d)$ tells us $X0$ is compact as a subspace of X. Density tells us $X0 = X$. The identity is then a homeomorphism and (X,G) is isomorphic to an isometric system; i.e., (X,G) is minimal and equicontinuous. [3]

5. PERIODIC POINTS AND MINIMAL SUBSYSTEMS

In [5], Devaney suggests three properties which define the essence of chaos. According to him, a dynamical system (i.e. a continuous map $T: X \rightarrow X$) should be called chaotic if it

(1) is topologically transitive,

(2) Has a dense set of periodic points,

(3) Has sensitive dependence on initial conditions.

It was first observed in [4] that these requirements are not independent. In fact, Banks et al. proved that the first two conditions imply the third (this is the content of Theorem 1.7). In [7], Glasner and Weiss derive this as a corollary of Theorem 1.8. They also produce a remarkably simple direct proof (their Corollary 1.4). Unfortunately, it is unclear to this author how to adapt the second argument to the case of a non-abelian acting group. Now we set out to prove Theorem 1.12. [4]

6. COROLLARY ALGORITHMS

Let g and Theorem 1.12 as and periodic set of points $(x,s\ ')$ (as generated by s _ group) for each $s \in S$ is dense value that if (X,G) sensitive, which form a narrower transitive $G\ X$ acts must be set on this condition that S Theorem 1.12 good drop Would be beneficial to. Unfortunately, the following example shows that this condition (or something like it) is inevitable. Let g be solvable groups and let $2Z\ Z\_Z/a$ & amp; $b\ Z$ and $Z/2Z$ generator factors respectively. then $s = \{a, AB\}$ is a generating set for g let g and single point of compact fiction action on x and g to I only if the left multiplication action on infinity system by fixing the point $x\ (X,G)$ is not transitive, but at least each s ∈ S the two orders so, every point $(X,s\ \_)$ system for at least. However, (X,G) are not sensitive dependence on initial conditions in fact, the only sensitive point is the point is infinite. All other points are isolated and equicontinuous. S is not good because it is not a counterexample to 1.12 theorems. Suppose that we make it good to add more generator S. for example, we can take $S = \{a, AB, [a,ab]\}$. The hypotheses are not met theorem: any $x \in x, lim\ |\ n\ | \rightarrow \infty\ [,ab]\ n;\ x = \infty$, which proves that only the lowest point $(X,[a,ab])\ \infty$ (dense explicitly). [4]

7. CONCLUSION

Its essential characteristic with chaotic theory: initial conditions, dynamic, sensitive dependence on the deterministic and non-linearity are the character of the real world. fern flora, its features and decide applications i.e. deterministic, dynamic, real world, complex and uncertain in a certain stage, they are not a decision maker decision making required for her job function should understand the real world. This paper discusses the chaos theory the applicability of your attributes and chaos theory in decision making process is to create application models are related to chaos decision represents. Addition, fern identification and classification in order to support the case and that are presented to prove the chaos and complexity constraints exist in application of decision making.

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